2. ENERGY AND MOMENTUM PRINCIPLE IN OPEN CHANNEL FLOW

2.1 basic energy equations

In the one dimensional analysis of steady open channel flow the energy equation in the form of Bernoulli equation is used. According to this equation the total energy at downstream section defers from the total energy at upstream section by an amount equal to the loss of energy between the sections.

It is known in elementary hydraulics that the total energy per unit weight of water of water in any Streamline passing through channel section may be expressed as the total head in meter of water, which is equal to the sum of the elevation above a datum, the pressure head, and the velocity head. For example, with respect to the datum plane, the total head *H* at a section 0 containing point *A* on a streamline of flow in a channel of large slope (Fig. 2.1) may be written as:-

$H = Z_A + d_A \cos\theta$ +— ……………………………………………………………..(eqn.2.1)

Where:-

Z_A is the elevation of point above the datum plane
d_A is the depth of point A below the free surface measured along the channel section
θ is the slope of the channel bed (bottom) and
V_A is the velocity head of the flow in the streamline passing through A.

In general, every streamline passing through a channel section will have different velocity head, owing to the nonuniform velocity distribution Actual in flow. Only in an ideal parallel flow of uniform velocity distribution can the velocity head be truly identical for all points on the cross section. In the case of gradually varied flow, however,· it may be assumed, for practical purposes, that the velocity heads for all points on the channel section are equal , and the energy coefficient may be used to Correct for the over-all effect of the non uniform velocity distribution.

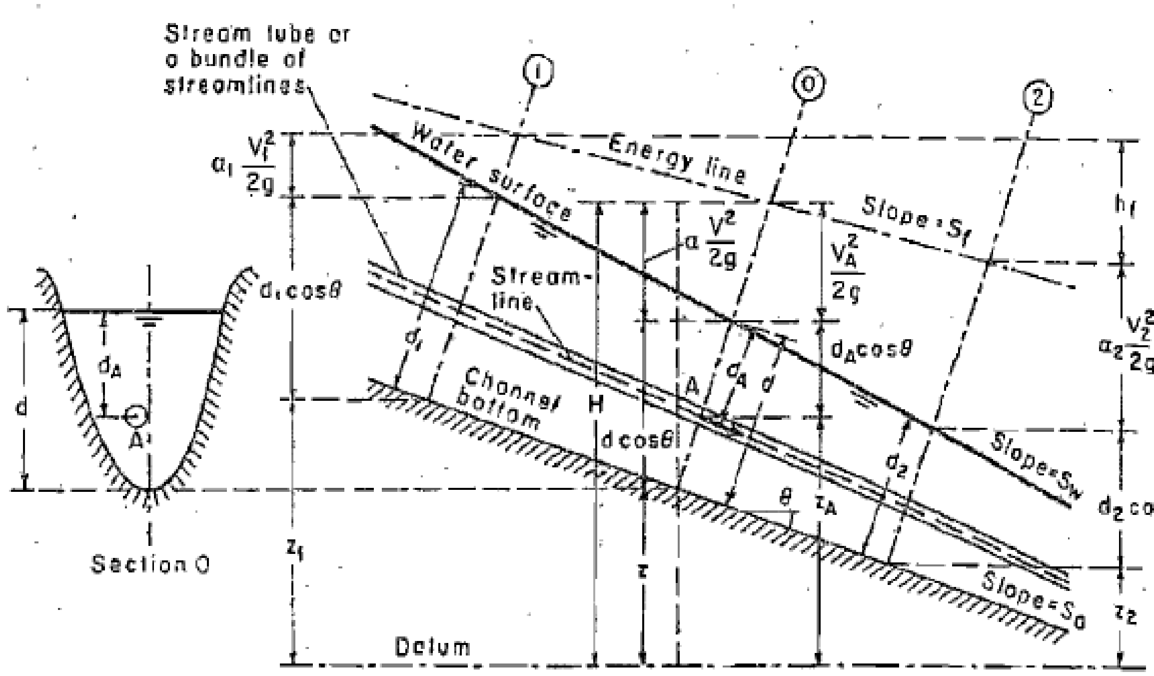


Fig 2.1 Energy in gradually varied open-channel flow.

Thus, the total energy at the channel section is

H = Z +dcosθ +α— **α** is the velocity distribution coefficient

For channels of small slope, θ = 0 thus, the total energy at the channel section is

H = Z + d + α—

According to the principle of conservation of energy, the total energy head at the upstream section 1 should be equal to the total energy head at the downstream section 2 plus the loss of energy *hf* between the two sections; or

$$Z_1 + d_1 \cos\theta + \alpha_1 — = Z_2 + d_2 \cos\theta + \alpha_2 + — + h_f$$

This equation applies to parallel or gradually varied flow. For a channel of small slope, it becomes:-

$$Z_1 + y_1 + \alpha_1 — = Z_2 + y_2 + \alpha_2 — + h_f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textit{(2.2)}$$

Either of these two equations is known as the **energy equation.** When **$\alpha_1 = \alpha_2 = 1$** and **$h_f = 0$**,it becomes,

— —

……………………………………………………….………(eqn.2.3)

This is the well-known **Bernoulli energy equation.**

2.2 Specific energy and critical depth

The total energy of a channel flow referred to a datum is given by equation below:

$$H = z + d\cos\theta + \alpha\frac{V^2}{2g}$$ α is the velocity distribution coefficient

If the datum coincides with the channel bed at the section, the resulting expression is known as specific energy and is denoted as E. thus

— for a channel of small slope and **α=1.**

— **for** $^{Q}/_{A}$ —— **..………………………………(eqn.2.4)**

For a channel of known geometry, E=*f(y*,Q), keeping Q constant it can be seen that, the specific energy in a channel section is a function of the depth of the flow only. The variation E with y is represented by a cubic parabola (Fig 2.2). it is seen that there are two positive roots for the equation of E indicating that any particular discharge Q_1 can be passed in a given channel at two depths and still maintain the same specific energy E. in the Figure 2.2 the ordinate PP’ represents the condition for a specific energy of E_1. the depth of flow can be either PR=y_1 or PR’=y_1’. These two possible depths have the same specific energy are known as ***alternate depths***. In the Figure 2.2, a line OS drawn such that E=y is the asymptote of the upper limb of the specific-energy curve. It may be noticed that the intercept P’R’ or P’R represents the velocity head of the two alternate depths, one (PR=y_1) is smaller and has a larger velocity head while the other (PR’=y_1’) has a larger depth and consequently a smaller velocity head.

The condition of minimum specific energy is known as ***the critical-flow condition*** and the corresponding depth y_c is known as ***critical depth***.

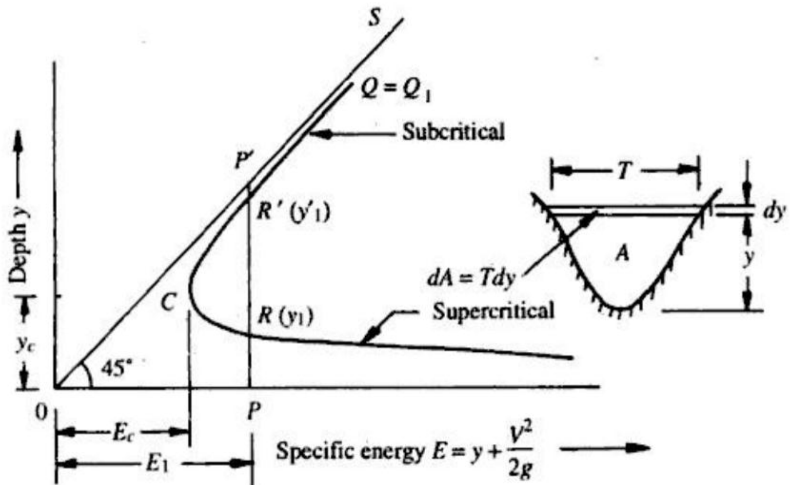


Fig. 2.2 Definition sketch of specific energy

Thus, at the critical state the two alternate depths apparently become one. When the depth of flow is greater than the critical depth, the velocity of flow is less than the critical velocity for the given discharge, and, hence, the flow is ***subcritica***l. When the depth of flow is less than the critical depth, the flow is ***supercritical***. Hence, y_1 is the depth of supercritical flow, and y_1' is the depth of subcritical flow.

At the critical depth, the specific energy is minimum. Thus differentiating (eqn 2.4) with respect to y (keeping Q constant) and equating to zero,

— —— But — top width, width of the channel at the water surface .

designating the critical-flow condition by the suffix ,**c**,.

—— ……………..…………………………………………………………………..(eqn.2.5)

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

- ✓ If an α value other than unity is used the above equation will be:

$$\frac{\alpha Q^2}{g} = \frac{A_c^3}{T_c}$$

- ✓ Critical flow condition is governed by the channel geometry and discharge (and α).
- ✓ If the Froude number is defined as:

$$F = \frac{V}{\sqrt{gA/T}}$$

- ✓ It is easy to see that at the critical flow $y=y_c$ $F=F_c=1$.

2.3 Critical depth for a variable discharge

In the above section the critical-flow condition was derived by keeping the discharge constant. The specific energy diagram can be plotted for different discharge $Q=Q_1$=constant (i=1,2,3...) in the figure, Q1 < Q2 < Q3 <.Q4. and is constant along the respective E vs y plot.

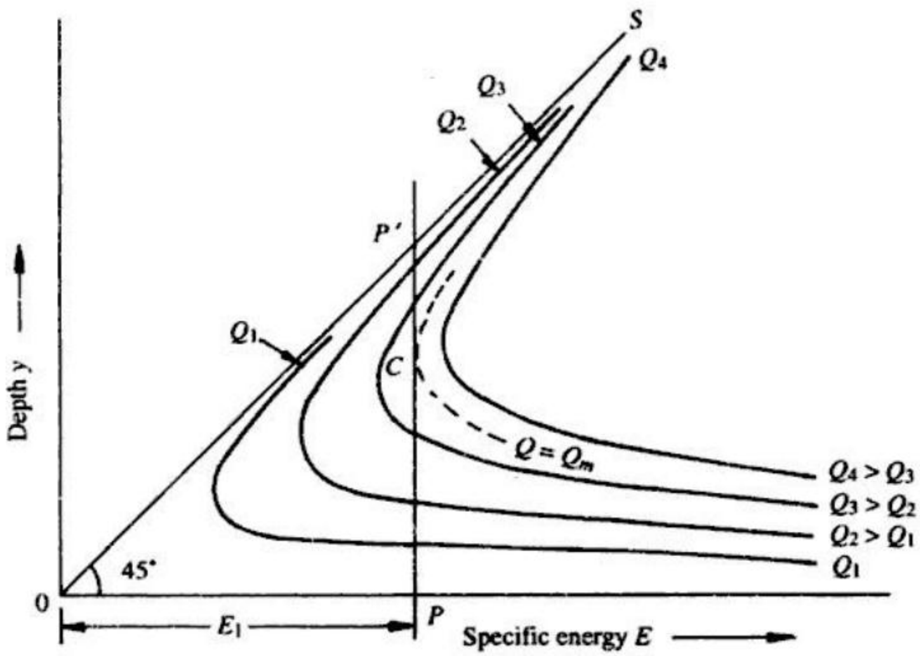


Fig 2.3 specific energy for varying discharge

Consider a section PP' in this plot, for the ordinate PP', E=E1=constant. Different Q curves give different intercepts. It is possible to imagine a value of $Q=Q_{max}$ at a point C at which the corresponding specific energy curve would be just tangent to the ordinate PP'. The dotted line indicating $Q=Q_{max}$ represents the maximum value discharge that can be passed in the channel while maintaining the specific energy at constant value E1.

——

$A\sqrt{2g\qquad}$

The condition for maximum discharge can be obtained by differentiating the above equation with respect to y and equating to zero while keeping E = constant.

$$\frac{dQ}{dy} = \sqrt{2g(E-y)}\frac{dA}{dy} - \frac{gA}{\sqrt{2g(E-y)}} = 0$$

$$\text{Putting } \frac{dA}{dy} = T \text{ and } \frac{Q}{A} = \sqrt{2g(E-y)}$$

—— = 1This is the same as the critical flow conditions. Hence, the critical flow condition also corresponds to the condition of maximum discharge in a channel for a fixed specific energy.

Section factor Z

The expression $A\sqrt{A/T}$ is a function of the depth y for a given channel geometry and is known as the section factor Z.

Those:

$Z=A\sqrt{A/T}$

At the critical flow condition $y=y_c$ and

$\sqrt{\quad /T_c}=$ —— at critical condition — → $\sqrt{g}$ — Thus: -

Z_c = $\frac{}{\sqrt{}}$ **……………………………………………………………..(eqn.2.6)**

2.4 Calculation of the Critical Depth

Using (eqn.2.5) expression for the critical depth in channel of various geometric shapes can be obtained as flows

A) Rectangular section

For rectangular section A=By and T=B hence the above expression becomes:-

—— — Or → — –

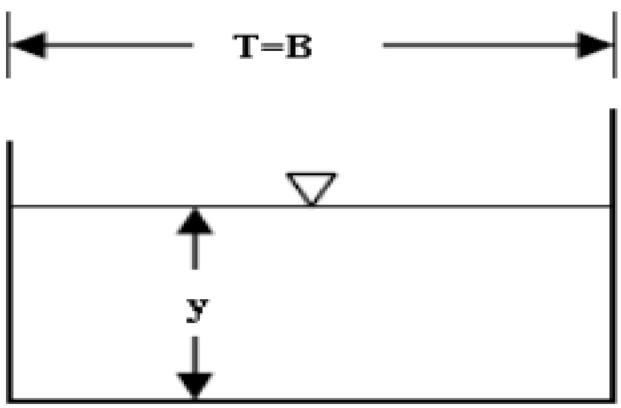


Fig 2.4 rectangular channel

Specific energy at critical depth E —

– **………………………………………………….. (eqn.2.7)**

The above equation is specific energy E is independent of the width of the channel.

Also if q = discharge per unit width = –.

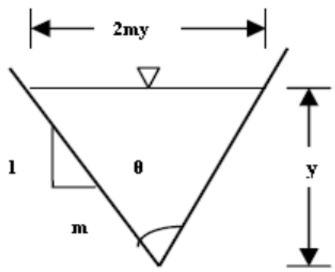
— i.e $y_c = \left(\frac{q^2}{g}\right)^{1/3}$ **……………………… (eqn.2.8)**

Since A/T the Froude number for a rectangular channel will be defined as

$\sqrt{gy}$ **……………………………………………………….. (eqn. 2.9)**

B) Triangular section

For a triangular having a side slope of m horizontal: 1 vertical and



$$\frac{Q^2T_c}{gA_c^3} = 1$$

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{m^3y_c^6}{2my_c} = \frac{m^2y_c^5}{2}$$

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$$

Fig 2.5 triangular channel

The specific energy at critical water depth.

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{Q^2}{2gA_c^2}$$

$$E_c = y_c + \frac{gm^2y_c^5}{2\times 2\times g\times m^2\times y_c^4}$$

$$E_c = y_c + \frac{y_c}{4} = 1.25y_c$$

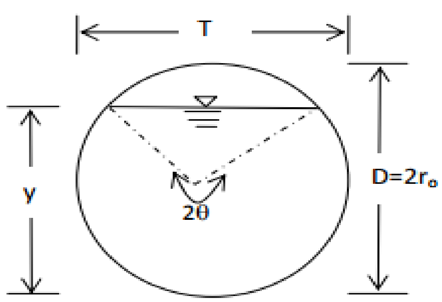
i.e. …………………………………… **(eqn.2.10)**

It is noted that eqn.2.10 is independent of the side slope ***m*** of the channel. Since A/T $Y/2$, the Froude number for a triangular channel is defined by:-

$$\frac{V\sqrt{2}}{\sqrt{gy}}$$ …………………………………... **(eqn.2.11)**

C) Circular Channel

Let D be the diameter of a circular channel and 2θ be the angel in radians subtended by the water surface at the center.



A = area of the flow section

= area of the sector + area of triangular portion

$$A = \frac{1}{2}r_o^2\,2\theta + \frac{1}{2}2r_o\sin(\pi-\theta)r_o\cos(\pi-\theta)$$

$$A = \frac{1}{2}r_o^2\,2\theta + \frac{1}{2}2r_o^2(2\sin(\pi-\theta)\cos(\pi-\theta))$$

$$A = \frac{1}{2}r_o^2\,2\theta - \frac{1}{2}2r_o^2(\sin 2\theta)$$

$$A = \frac{1}{2}r_o^2(2\theta - \sin 2\theta)$$

$$\boxed{A = \frac{1}{8}D^2(2\theta - \sin 2\theta)}$$ $$\boxed{T = D\sin\theta}$$

and $$2\theta = 2\cos^{-1}\left(1-\frac{2y}{D}\right) = f(y/D)$$

Substituting these in Eq. (2.4a) yields

$$\frac{Q^2}{g} = \frac{\left[\frac{D^2}{8}(2\theta_c - \sin 2\theta_c)\right]^3}{D\sin\theta_c} \quad \text{.......................................(2.12)}$$

Since explicit solutions for y_c cannot be obtained from Eq. (2.12), a non-dimensional representation of Eq. (2.12) is obtained as

$$\frac{Q}{\sqrt{gD^5}} = \frac{0.044194\ (2\theta_c - \sin 2\theta_c)^{3/2}}{(\sin\theta_c)^{1/2}} = f(y_c/D) \quad \text{.....................(2.13)}$$

This function is evaluated and is given in Table 2A.1 of Appendix 2A at the end of this chapter as an aid for the estimation of y_c.

Since $A/T = fn\left(\frac{y}{D}\right)$, the Froude number for a given Q at any depth y will be

$$F = \frac{V}{\sqrt{g(A/T)}} = \frac{Q}{\sqrt{g(A^3/T)}} = fn(y/D)$$

C) Trapezoidal channel

For a trapezoidal channel having a bottom width of ***B*** and side slopes of ***m*** horizontal: 1 vertical (fig (2.7)).

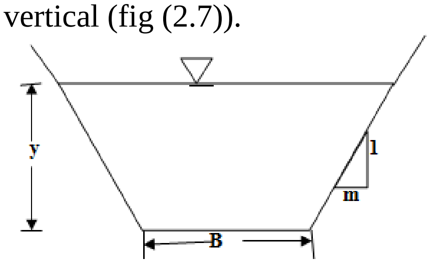


Fig 2.7 trapezoidal channel

Area $A = (B + my)y$

and Top width $T = (B + 2my)$

At the critical flow

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{(B + my_c)^3 y_c^3}{(B + 2my_c)} \quad \text{..........................(2.14)}$$

Here also an explicit expression for the critical depth y is not possible .the non-dimensional representation of Eqn. (2.14) facilitates the solution of y by the aid of tables or graphs. Rewriting the right- hand side of Eqn. (2.14) as

$$\frac{(B + my_c)^3\, y_c^3}{B + 2my_c} = \frac{B^3\left(1 + \frac{my_c}{B}\right)^3 y_c^3}{B\left(1 + \frac{2my_c}{B}\right)}$$

$$= \frac{B^5}{m^3}\frac{(1+\zeta_c)^3\,\zeta_c^3}{(1+2\zeta_c)} \qquad \text{where } \zeta_c = \frac{m\,y_c}{B}$$

gives $$\frac{Q^2 m^3}{g B^5} = \frac{(1+\zeta_c)^3 \zeta_c^3}{(1+2\zeta_c)} \quad \text{...(2.15)}$$

or $$\frac{Q m^{3/2}}{\sqrt{g}\, B^{5/2}} = \psi = \frac{(1+\zeta_c)^{3/2} \zeta_c^{3/2}}{(1+2\zeta_c)^{1/2}} \quad \text{............................(2.16)}$$

Equation 2.16 can easily be evaluated for various value of ξ_c and plotted as ψ vs ξ_c it may be noted that if $\alpha > 1$, ψ can be defined as

$$\psi = \left(\frac{\alpha Q^2 m^3}{g B^5}\right)^{1/2} \quad \text{....................................(2.17)}$$

One such plot, shown in Fig. 2.7, is very helpful in quick estimation of critical depth and other parameters related to the critical-flow condition in trapezoidal channels. Table 2A.2 which gives values of ψ for various ζ_c values is useful for constructing a plot of ψ vs ζ_c as in Fig. 2.7 on a larger scale.

Since $A/T = \dfrac{(B+my)y}{(B+2my)} = \dfrac{\left(1+\frac{my}{B}\right)y}{\left(1+2\frac{my}{B}\right)}$ **the Froude number at any depth y is**

$$F = \frac{V}{\sqrt{gA/T}} = \frac{Q/A}{\sqrt{gA/T}} = fn\ (my/B) \text{ for a given discharge } Q.$$

Further the specific energy at critical depth, E_c is a function of $(m\,y_c/B)$ and it can be shown that (Problem 2.7)

$$\frac{E_c}{y_c} = \frac{1}{2}\frac{(3+5\zeta_c)}{(1+2\zeta_c)}$$

where $\zeta_c = \dfrac{my_c}{B}$

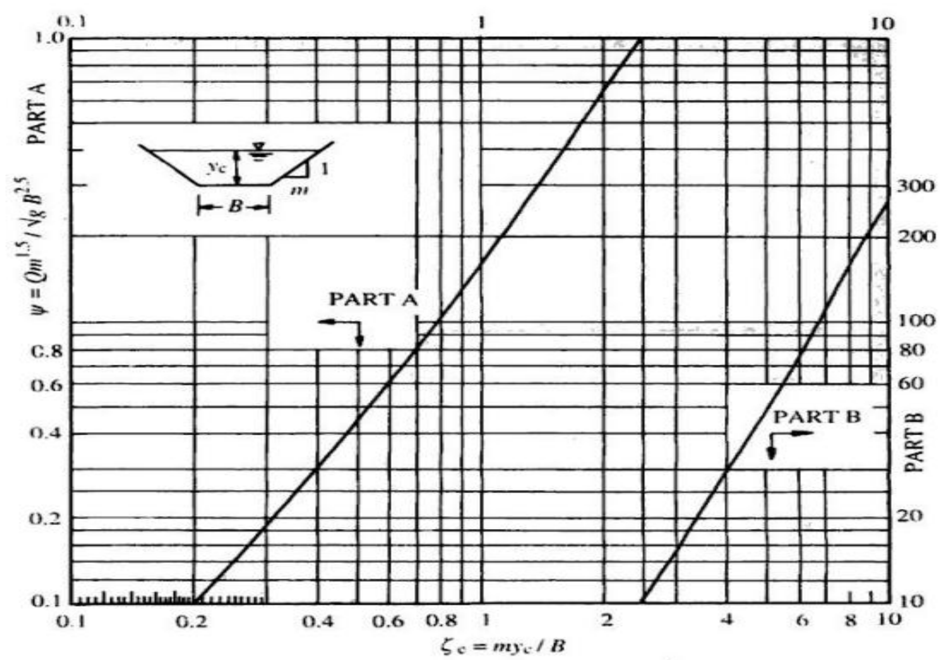


Fig. 2.7 Variation of ψ with ζ_c

2.5 Channel transitions

2.5.1 Channel with a Hump /rise in bed level/

A. subcritical flow

Consider a horizontal, frictionless rectangular channel of width B carrying Q at a depth y1. Let the flow be subcritical. At section 2 a smooth hump of height ΔZ is built on the floor. Since there is no energy losses between section 1 and 2, and construction of a hump causes the specific energy at section 2 to decrease by ΔZ. The specific energy at section 1 and 2 are given by:

— **i.e.**

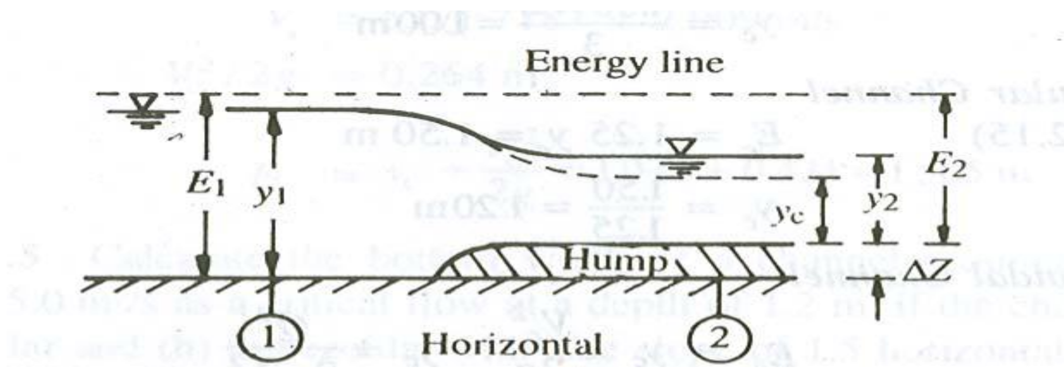


Figure 2.8 channel transition with a hump

Since the flow is subcritical, the water surface will drop due to a decrease in the specific energy. In figure 2.9, the water surface which was at P at section 1 will come down to point R at section 2. The depth y_2 will be given by:-

$$E_2 = y_2 + \frac{V_1^2}{2g} = y_2 + \frac{Q^2}{2gB^2 y_2^2}$$

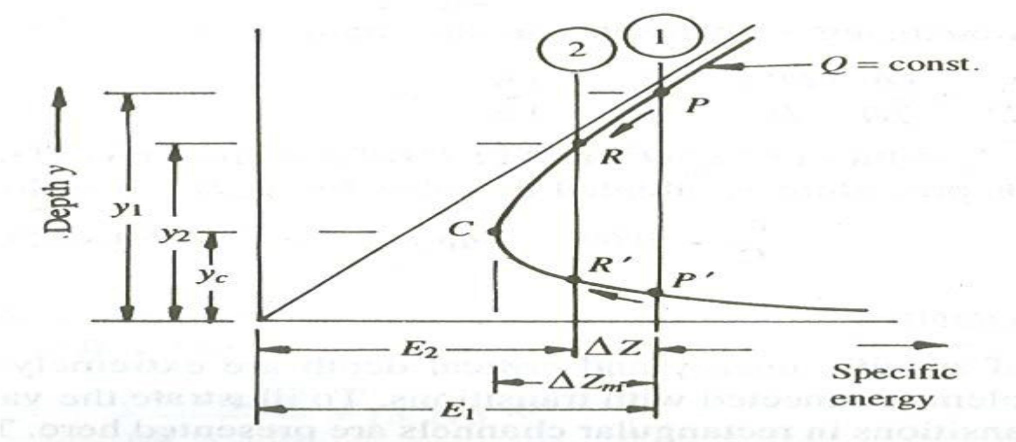


Fig 2.9 Specific energy diagram

It is easy to see from fig 2.9 that as the value of ΔZ is increased, the depth at section 2, will decrease. The minimum depth is reached when the point R coincides with C, the critical depth point.

At this point the hump height will be maximum, say ΔZ_m , $y_2=y_c$ critical depth and $E_2=E_c$. the condition at ΔZ_m is given by- the relation:-

$$E_1 - \Delta Z_m = E_2 = E_c = y_c + \frac{Q^2}{2gB^2 y_c^2}$$

For the hump height greater than ΔZ_m the flow is not possible with the given specific energy. The upstream depth has to increase to cause an increase in specific energy at section 1. if this modified depth is represented by y_1', then

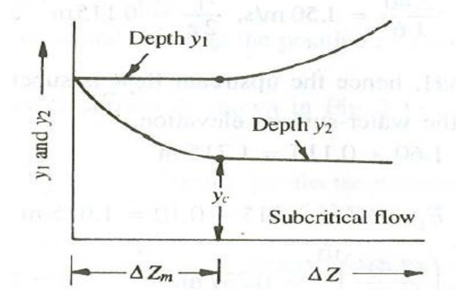
$$E_1^{'} = y_1^{'} + \frac{Q^2}{2gB^2 y_1^{'2}} \quad \text{with } \{E_1' > E_1 \text{ and } y_1' > y_1\}$$

At section 2 the flow will continue at the minimum pecific energy level, i.e. at the critical condition. At this condition $y_2 = y_c$ and

$$E_1^{'} - \Delta Z_m = E_2 = E_c = y_c + \frac{Q^2}{2gB^2 y_c^2}$$

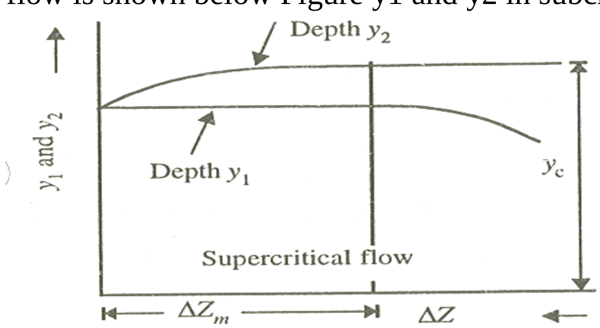
- ✓ When $0<\Delta Z<\Delta Z_m$ the upstream water level remains stationary at y_1 while the depth of flow at section 2 decrease with ΔZ a minimum value of y_c at $\Delta Z=\Delta Z_m$.
- ✓ With further increase of ΔZ for $\Delta Z>\Delta Z_m$, y_1 will change to y_1' while y_2 will continue to remain at y_c'.

Figue variation of y1 and y2 in subcritical flow over a hump.



B. supercritical flow

If y_1 is in the supercritical flow regime, fig 2.9 shows that the depth of flow increase due to the reduction of specific energy. In figure 2.9 point P' corresponds to y_1 and point R' to a depth at section 2. Up to the critical depth, y_2 increases to reach y_c at $\Delta Z=\Delta Z_m$. For $\Delta Z>\Delta Z_m$, the depth over the hump $y_2=y_c$ will remain constant and the upstream depth y_1 will change. It will decrease to have a higher specific energy E_1'. The variation of the depths y1 and y2 with ΔZ in the super critical flow is shown below Figure y1 and y2 in subcritical flow over the hump.



2.5.2 Transition with change in width

a. Subcritical flow in a width construction

Consider a friction less horizontal channel of width B_1 carrying a discharge Q at a depth y_1 as in figure 2.10. At section 2 the channel width has been constricted to B_2 by a smooth transition. Since there are no losses involved and since the bed elevation at section 1 and 2 are the same, the specific energy at section 1 and 2 are the same.

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2gB_1^2 y_1^2} \text{ and } E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{Q^2}{2gB_2^2 y_2^2}$$

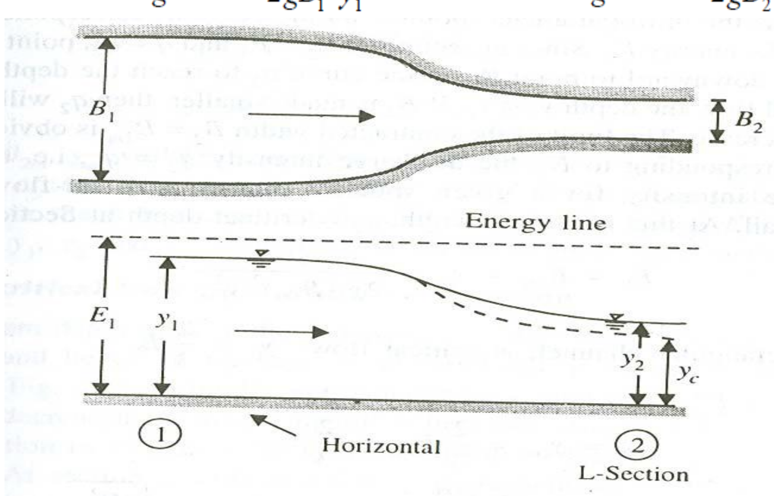


Fig 2.10 transition with width construction

It is convenient to analyses the flow in terms of the discharge intensity –. At section one — and at section two $q_2 = —$ Since $B_2 < B_1$, $q_2 > q_1$. The specific energy diagram fig 2.11 drawn with discharge intensity as the third parameter, point P on the curve q_1 corresponds to a depth y_1 and specific energy E_1.

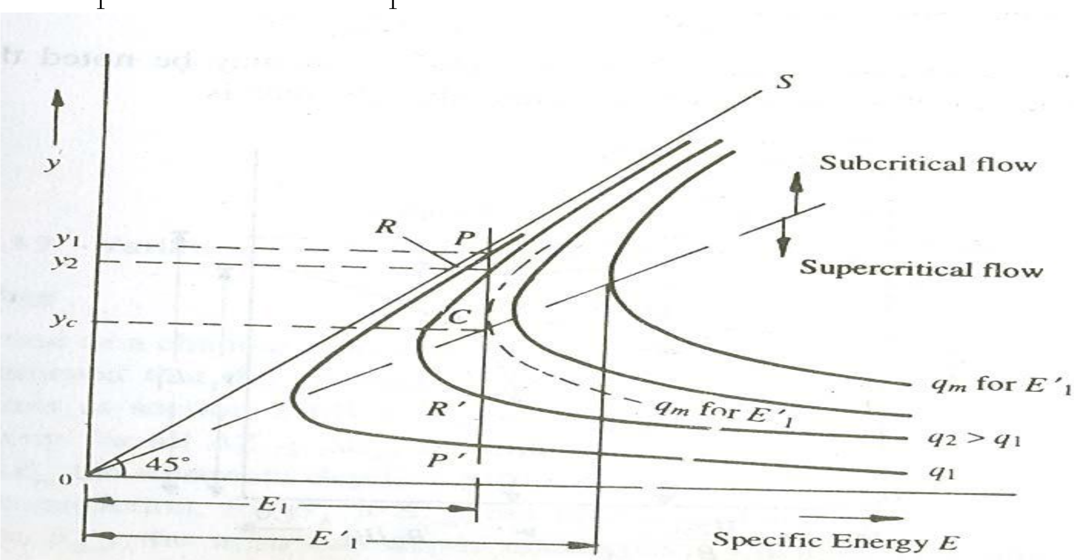


Figure 2.11 specific energy diagram

Since at section 2, E_2=E_1 and q=q_2, point P will move vertically downward to point R on the curve q_2 to reach the depth y_2. Those in subcritical flow the depth $y_2 < y_1$. If B_2 is made smaller, then q_2 will increase and y_2 will decrease. The limit of the contracted width B_2=B_{2m} is obviously reached when corresponding to E_1, the discharge intensity q_2=q_{2m}, i.e. the maximum discharge intensity for a given specific energy (critical flow condition) will prevail. At this minimum width, y_2 = critical depth at section 2, y_{cm} and.

$$E_1 = E_{cm} = y_{cm} + \frac{Q^2}{2gB_{cm}^2 y_{cm}^2}$$

For a rectangular channel, at critical flow $y_c = \frac{2}{3}E_c$

Since E_1=E_{cm}

$y_2 = y_{cm} = \frac{2}{3}E_{cm} = \frac{2}{3}E_1$ and

And $\frac{Q^2}{g} = \frac{A_c^3}{T_c}$ →

$$y_c = \left(\frac{Q^2}{B_{2m}^2}\right)^{1/3} \text{ or } B_{2m} = \sqrt{\frac{Q^2}{gy_{cm}^3}}$$

i.e

$$B_{2m} = \sqrt{\frac{27Q^2}{8gE_1^3}}$$

If B_2<B_{2m}, the discharge intensity q_2 will be larger than q_m the maximum discharge intensity consist ant with E1. The flow will not be possible with the given upstream condition. Therefore, the upstream depth will have to increase to y_1' so that a new specific energy is formed which will just be sufficient to cause critical flow at section 2.

✓ The new critical depth at section 2 for a rectangular channel is:

$$y_{c2} = \sqrt{\frac{Q^2}{B_2^2 g}} = \left(\frac{q_2^2}{g}\right)^{1/3} \text{ and}$$

$$E_{c2} = y_{c2} + \frac{V_{c2}^2}{2g} = 1.5y_c$$

Since B_2 <B_{2m} , y_{c2} will be larger than y_{cm} further E1 '=E_{c2} =1.5y_{c2}. thus even though critical flow prevails for all B_2 <B_{2m}, the depth at section 2 is not constant as in the hump case but increase as y_1 ' and hence E_1' rises. The variation of y_1 , y_2 and E with B_2/ B_1 is shown schematically in figure 2.12.

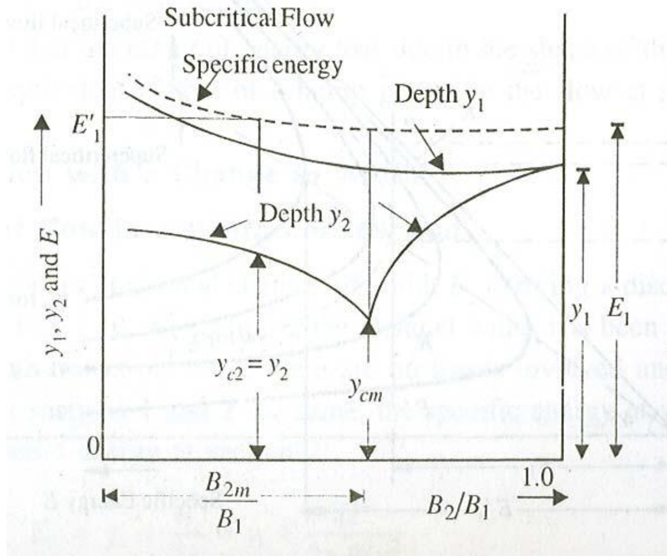


Figure 2.12 Variation of y_1 and y_2 in subcritical flow in a width constriction.

B. supercritical flow in width constriction

If the upstream depth is supercritical flow regime, a reduction of the flow width and hence increase in discharge intensity cause a rise in depth y_2. In figure 2.11, point P' corresponds to y_1 and point R' to y_2 as the width B_2 is decreased, R' moves up till it becomes critical at $B_2=B_{2m}$ any further reduction in B_2 causes the upstream depth to decrease to y_1' so that E_1 rises to E_2'. At section 2, critical depth y_c' corresponds to the new specific energy E_1' will prevail. The variation of y_1, y_2 and E with B_2/B_1 in supercritical regime is shown below.

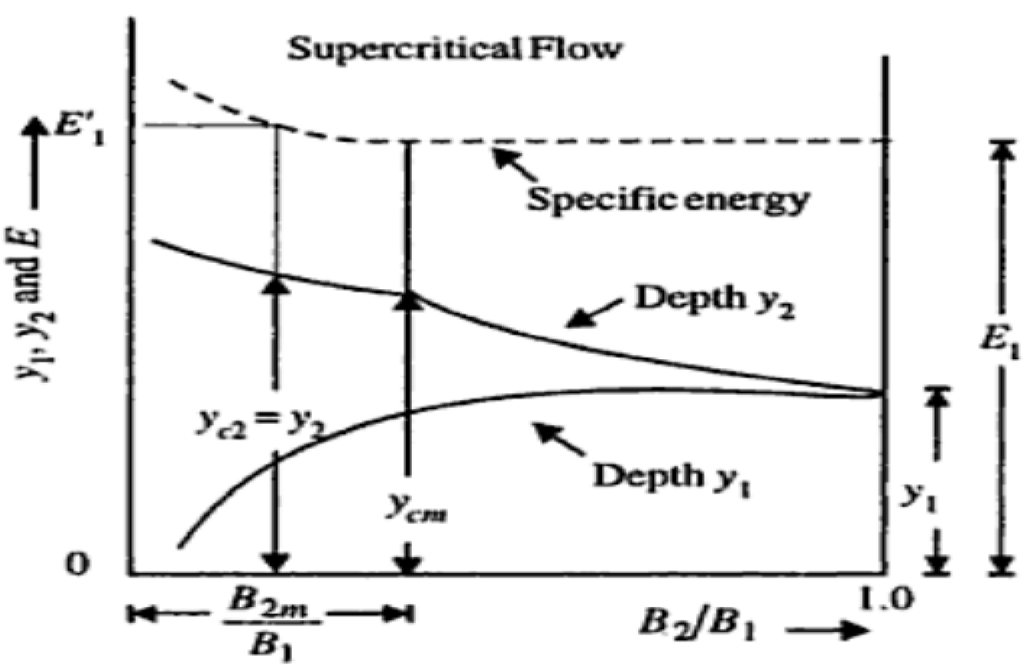


Fig.2.13 Variation of y_1 and y_2 in super-critical flow in width constriction

2.6 Momentum Principle in Open Channel flows

The momentum equation commonly used in most of the open channel flow problems is a linear-momentum equation. This equation states that the algebraic sum of all external forces acting in a given direction on a fluid mass equals the time rate of change of linear-momentum of the fluid mass in that direction. In a steady flow the rate of change of momentum in a given direction will be equal to the net flux of momentum in that direction.

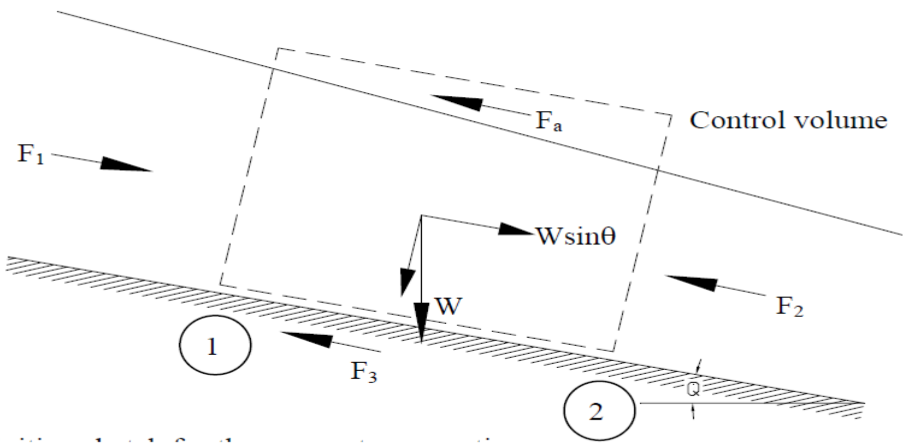


Figure 2.13 definition sketch for the momentum equation

The figure shows a control volume (a volume fixed in space) bounded by section 1 and 2, the boundary and a surface laying above the free surface. The various forces acting on the control volume in the longitudinal direction are:

a) Pressure force acting on the control surfaces, F1 and F2.
b) Tangential force on the bed F3,
c) Body force, i.e the component of the weight of the fluid in the longitudinal direction F4.
d) F_a is the air resistance at the free surface.

$$\sum F_x = W \sin\theta + F_1 - F_2 - F_3 - F_a = M_2 - M_1$$

Here F_1 and F_2 are hydrostatic pressures on the section 1 and 2, W is the weight of the control volume, θ is the slope of the bed with the horizontal, F3 is the boundary friction over the length ΔX and F_a is the air resistance at the free surface. Generally speaking F_a is negligible and is customary to neglect F3 also when ΔX is small. In which M_1 and M_2 are momentum flux entering and leaving the control Volume $\beta\rho Q V_1$ and β .

$$\sum F_x = W \sin\theta + F_1 - F_2 - F_3 - F_a = Q\rho(\beta_2 v_2 - v_1\beta_1)$$

In practical applications of the momentum equation, the proper identification of the control volume and the various forces acting on it are very important. The momentum equation is practically useful tool in analyzing rapidly varied flow (RVF) situation where energy loss are complex and cannot be easily estimated.

With assumption that θ=0 and $\beta_1=\beta_2 = 1$ and very small tangential force the above equation becomes:

$$\gamma Z_1 A_1 - \gamma Z_2 A_2 = \rho Q(v_2 - v_1)$$

Where Z_1 and Z_2 are the distance to the centroid of respective cross sectional flow areas A_1 and A_2 .

$$y_1 A_1 - y\ \ A_2 = \frac{Q^2}{g}\left(\frac{1}{A_2} - \frac{1}{A_1}\right)$$

$\frac{Q_1}{gA_1} + Z_1 A_1 = \frac{Q_2}{gA_2} + Z_2 A_2$ is known as specific momentum of force function.

Hydraulic jump

Hydraulic jump is formed when ever supercritical flow changes to subcritical flow, at the jump location there is a sharp increase in water surfaces and a considerable amount of energy is dissipated due to turbulence. At present we are interested in developing a relationship between the flow depth and flow velocities upstream and downstream of the jump of the flow. Upstream and downstream depths of the flow are called ***sequent depths or conjugate depths.***

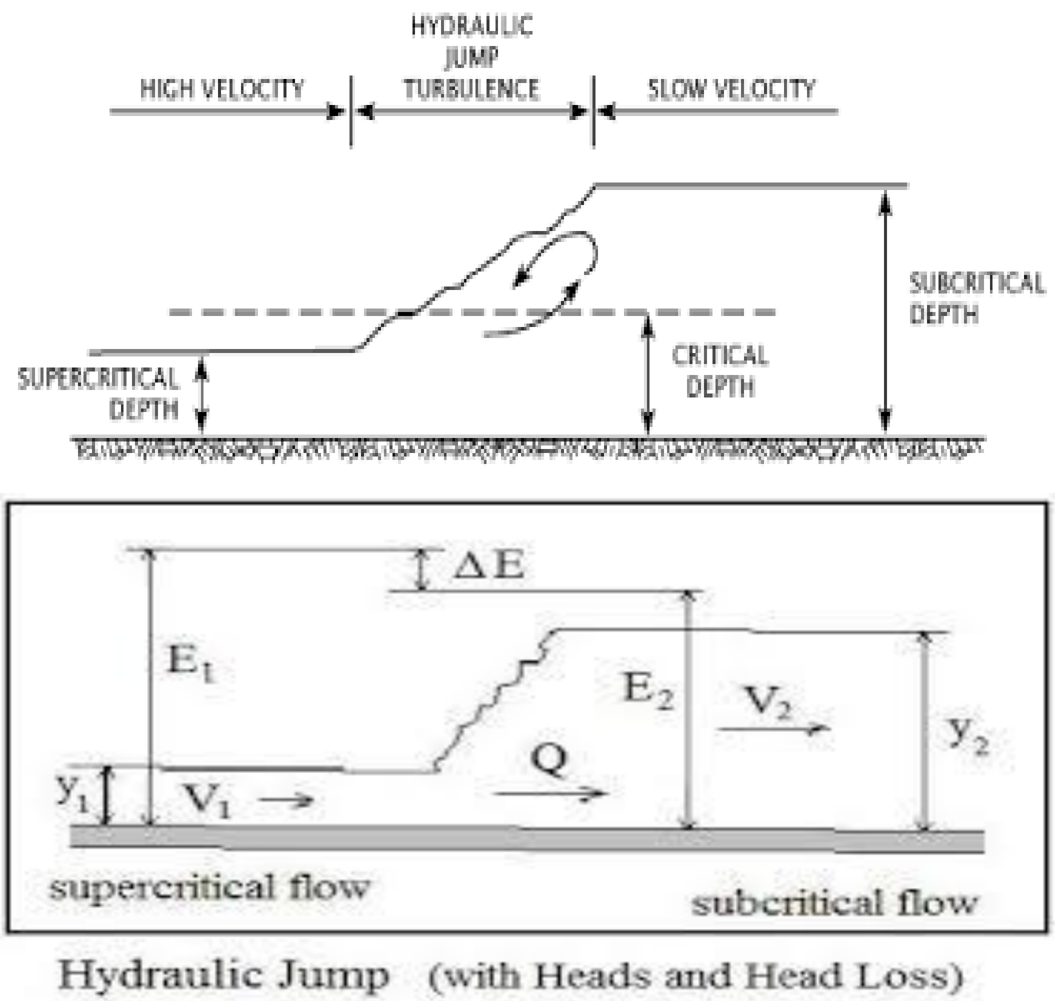


Figure 2.14 hydraulic jump

To simplify the derivation: we will consider a rectangular horizontal channel, since the amount of energy losses in the jump is not known in advance we can't apply the energy equation directly. However since the length of the jump is very short, the losses due to the shear at the channel bottom and sides are small as compared to the pressure force and may be neglected. In addition since the channel is horizontal, the component of the weight of water in the flow downstream direction is zero.

$$F_1 - F_2 = Q\rho(\beta_2 v_2 - v_1\beta_1) \quad \text{momentum correction factor} = 1$$

$$\frac{1}{2}\rho g y_1^2 - \frac{1}{2}\rho g y_2^2 = Q\rho(v_2 - v_1) \qquad Q = A_1 v_1 = A_2 v_2 \text{ consider a unit width rectangular channel}$$

$$\frac{1}{2}g(y_1^2 - y_2^2) = q(v_2 - v_1)$$

$$\frac{1}{2}g(y_1 - y_2)(y_1 + y_2) = q\left(\frac{q}{y_2} - \frac{q}{y_1}\right)$$

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g}$$

$$y_1 y_2 (y_1 + y_2) = \frac{2v_1^2 y_1^2}{g} \quad \text{for rectangular section } F_1 = \frac{v_1}{\sqrt{g y_1}} \qquad F_1^2 = \frac{v_1^2}{g y_1}$$

$$\frac{y_2}{y_1^2}(y_1 + y_2) = 2F_1^2 \qquad -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{y_2}{y_1} + \left(\frac{y_2}{y_1}\right)^2 = 2F_1^2 \quad \text{solving for y2/y1}$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8F_1^2}\right)$$

Table 2A.2 Values of ψ for Computation of Critical Depth in Trapezoidal Channels

ζ_c	ψ	ζ_c	ψ	ζ_c	ψ	ζ_c	ψ
0.010	0.001005	0.330	0.225681	0.890	1.308445	1.450	3.390540
0.015	0.001851	0.340	0.237258	0.900	1.336334	1.460	3.437866
0.020	0.002857	0.350	0.249103	0.910	1.364544	1.470	3.485566
0.025	0.004003	0.360	0.261215	0.920	1.393075	1.480	3.533642
0.030	0.005276	0.370	0.273595	0.930	1.421927	1.490	3.582093
0.035	0.006665	0.380	0.286245	0.940	1.51103	1.500	3.630922
0.040	0.008164	0.390	0.299164	0.950	1.480602	1.501	3.680128
0.045	0.009767	0.400	0.312353	0.960	1.510426	1.502	3.729714
0.050	0.011469	0.410	0.325813	0.970	1.540577	1.503	3.779679
0.055	0.013267	0.420	0.339545	0.980	1.571054	1.540	3.830025
0.060	0.015156	0.430	0.353550	0.990	1.601859	1.550	3.880752
0.065	0.017134	0.440	0.367827	1.000	1.632993	1.560	3.931862
0.070	0.019199	0.450	0.382379	1.010	1.664457	1.570	3.983355
0.075	0.021348	0.460	0.397206	1.020	1.696253	1.580	4.035232
0.080	0.023580	0.470	0.412308	1.030	1.728380	1.590	4.087495
0.085	0.025893	0.480	0.427687	1.040	1.760840	1.600	4.140143
0.090	0.028285	0.490	0.443344	1.050	1.793634	1.610	4.193178
0.095	0.030756	0.500	0.459279	1.060	1.826764	1.620	4.246602
0.100	0.033304	0.510	0.475494	1.070	1.860229	1.630	4.300414
0.105	0.035928	0.520	0.491989	1.080	1.894031	1.640	4.354615
0.110	0.038627	0.530	0.508765	1.090	1.928171	1.650	4.409207
0.115	0.041401	0.540	0.525824	1.100	1.962651	1.660	4.464190
0.120	0.044247	0.550	0.543165	1.110	1.997473	1.670	4.519566
0.125	0.047167	0.560	0.560791	1.120	1.032630	1.680	4.575335
0.130	0.050159	0.570	0.578702	1.130	1.068132	1.690	4.631497
0.135	0.053222	0.580	0.596899	1.140	1.103977	1.700	4.688055
0.140	0.056357	0.590	0.615383	1.150	1.140166	1.710	4.745008
0.145	0.059561	0.600	0.634155	1.160	1.176700	1.720	4.802358
0.150	0.062836	0.610	0.653216	1.170	1.213580	1.730	4.860105

ζ_c	ψ	ζ_c	ψ	ζ_c	ψ	ζ_c	ψ
0.155	0.066181	0.620	0.672568	1.180	2.250806	1.740	4.918251
0.160	0.069595	0.630	0.692210	1.190	2.288381	1.750	4.976796
0.165	0.073078	0.640	0.712145	1.200	2.326304	1.760	5.035741
0.170	0.076630	0.650	0.732373	1.210	2.364577	1.770	5.095087
0.175	0.080250	0.660	0.752894	1.220	2.403200	1.780	5.154835
0.180	0.083939	0.670	0.773711	1.230	2.442176	1.790	5.214986
0.185	0.087695	0.680	0.794825	1.240	2.481504	1.800	5.275540
0.190	0.091519	0.690	0.816235	1.250	2.521185	1.810	5.336498
0.195	0.095411	0.700	0.837944	1.260	2.561222	1.820	5.397862
0.200	0.099369	0.710	0.859952	1.270	2.501613	1.830	5.459631
0.205	0.103396	0.720	0.882260	1.280	2.642361	1.840	5.521808
0.210	0.107489	0.730	0.904869	1.290	2.683467	1.850	5.584393
0.215	0.111649	0.740	0.927781	1.300	2.724931	1.860	5.647386
0.220	0.115876	0.750	0.950997	1.310	2.766755	1.870	5.710789
0.225	0.120169	0.760	0.974516	1.320	2.808938	1.880	5.774602
0.230	0.124530	0.770	0.998342	1.330	2.851483	1.890	5.838826
0.235	0.128957	0.780	1.022473	1.340	2.897660	1.900	5.903462
0.240	0.133450	0.790	1.046912	1.350	2.937660	1.910	5.968511
0.245	0.138010	0.800	1.071660	1.360	2.981295	1.920	6.033974
0.250	0.142636	0.810	1.096717	1.370	3.025294	1.930	6.099851
0.260	0.152088	0.820	1.122085	1.380	3.069659	1.940	6.166144
0.270	0.161805	0.830	1.147765	1.390	3.114391	1.950	6.232853
0.280	0.171787	0.840	1.173757	1.400	3.159491	1.960	6.299979
0.290	0.182034	0.850	1.200063	1.410	3.204960	1.970	6.367523
0.300	0.192547	0.860	1.226684	1.420	3.250789	1.980	6.435486
0.310	0.203326	0.870	1.253620	1.430	3.297007	1.990	6.503868
0.320	0.214370	0.880	1.280874	1.440	3.343587	2.000	6.572671

Table 2A.1 Elements of Circular Channels*

y/D	2θ (radians)	A/D²	P/D	T/D	Z/D^2.5	AR^2/3/D^8/3
0.01	0.40067E+00	0.13293E-02	0.20033E+00	0.19900E+00	0.10865E-03	0.46941E-04
0.02	0.56759E+00	0.37485E-02	0.28379E+00	0.28000E+00	0.43372E-03	0.20946E-03
0.03	0.69633E+00	0.68655E-02	0.34817E+00	0.34117E+00	0.97392E-03	0.50111E-03
0.04	0.80543E+00	0.10538E-01	0.40272E+00	0.39192E+00	0.17279E-02	0.92878E-03
0.05	0.90205E+00	0.14681E-01	0.45103E+00	0.43589E+00	0.26944E-02	0.14967E-02
0.06	0.98987E+00	0.19239E-01	0.49493E+00	0.47497E+00	0.38721E-02	0.22078E-02
0.07	0.10711E+00	0.24168E-01	0.53553E+00	0.51029E+00	0.52597E-02	0.30636E-02
0.08	0.11470E+01	0.29435E-01	0.57351E+00	0.54259E+00	0.68559E-02	0.40652E-02
0.09	0.12188E+01	0.35012E-01	0.60939E+00	0.57236E+00	0.86594E-02	0.52131E-02
0.10	0.12870E+01	0.40875E-01	0.64350E+00	0.60000E+00	0.10669E-01	0.65073E-02
0.11	0.13523E+01	0.47006E-01	0.67613E+00	0.62578E+00	0.12883E-01	0.79475E-02
0.12	0.14150E+01	0.53385E-01	0.70748E+00	0.64992E+00	0.15300E-01	0.95329E-02
0.13	0.14755E+01	0.59999E-01	0.73773E+00	0.67261E+00	0.17920E-01	0.11263E-01
0.14	0.15340E+01	0.66833E-01	0.76699E+00	0.69397E+00	0.20740E-01	0.13136E-01
0.15	0.15908E+01	0.73875E-01	0.79540E+00	0.71414E+00	0.23760E-01	0.15151E-01
0.16	1.67607	0.08111	0.82303	0.73321	0.02698	0.01731
0.17	1.69996	0.08854	0.84998	0.75127	0.03039	0.01960
0.18	1.75260	0.09613	0.87630	0.76837	0.03400	0.02203
0.19	1.80411	0.10390	0.90205	0.87460	0.03781	0.02460
0.20	1.85459	0.11182	0.92730	0.80000	0.04181	0.02729
0.21	1.90414	0.11990	0.95207	0.81462	0.04600	0.03012
0.22	1.95282	0.12811	0.97641	0.82849	0.05038	0.03308
0.23	2.00072	0.13647	1.00036	0.84167	0.05495	0.03616
0.24	2.04789	0.14494	1.02395	0.85417	0.05971	0.03937
0.25	2.09440	0.15355	1.04720	0.86603	0.06465	0.04270
0.26	2.14028	0.16226	1.07414	0.87727	0.06979	0.04614
0.27	2.18560	0.17109	1.09280	0.88792	0.07510	0.04970
0.28	2.23040	0.18002	1.11520	0.89800	0.08060	0.05337
0.29	2.27470	0.18905	1.13735	0.90752	0.08628	0.05715
0.30	2.31856	0.19817	1.15928	0.91652	0.09215	0.06104
0.31	2.36200	0.20738	1.18100	0.92499	0.09819	0.06503
0.32	2.40506	0.21667	1.20253	0.93295	0.10441	0.06912
0.33	2.44776	0.22603	1.22388	0.94343	0.11081	0.07330
0.34	2.49013	0.23547	1.24507	0.94742	0.11739	0.07758
0.35	2.53221	0.24498	1.26610	0.95394	0.12415	0.08195
0.36	2.57400	0.25455	1.28700	0.66000	0.13108	0.08641
0.37	2.61555	0.26418	1.30777	0.96561	0.13818	0.09095
0.38	2.65686	0.27386	1.32843	0.97077	0.14546	0.09557

*The notations '$E+a$' represents 10^a and '$E-a$' represents 10^{-a}. Thus for example

0.13523E + 01 = 1.3523

0.47006E − 01 = 0.047006

Note: The following correlation equation due to Swamee[4] can be used for quick estimation of critical depths in circular channels with errors less than 1.25%.

$$\frac{y_c}{D} = \left[0.77 F_D^{-6} + 1.0\right]^{-0.085}$$

where $$F_D = \frac{Q}{D^2\sqrt{gD}} = Z/D^{2.5}$$

y/D	2θ (radians)	A/D^2	P/D	T/D	$Z/D^{2.5}$	$AR^{2/3}/D^{8/3}$
0.39	2.69796	0.28359	1.34898	0.97550	0.15291	0.10027
0.40	2.73888	0.29337	1.36944	0.97980	0.16053	0.10503
0.41	2.77962	0.30319	1.38981	0.98367	0.16832	0.10987
0.42	2.82021	0.31304	1.41011	0.98712	0.17629	0.11477
0.43	2.86067	0.32293	1.43033	0.99015	0.18442	0.11973
0.44	2.90101	0.33284	1.45051	0.99277	0.19272	0.12475
0.45	2.94126	0.34278	1.47063	0.99499	0.20120	0.12983
0.46	2.98142	0.35278	1.49079	0.99699	0.20984	0.13495
0.47	3.02152	0.36272	1.51076	0.99820	0.21865	0.14011
0.48	3.06157	0.37270	1.53079	0.99920	0.22763	0.14532
0.49	3.10159	0.38370	1.55080	0.99980	0.23677	0.15057
0.50	3.14159	0.39270	1.57080	0.00000	0.24609	0.15584
0.51	3.18160	0.40270	1.59080	0.99980	0.25557	0.16115
0.52	3.22161	0.41269	1.61081	0.99920	0.26523	0.16648
0.53	3.26166	0.42268	1.63083	0.99820	0.27505	0.17182
0.54	3.30176	0.43266	1.65088	0.99679	0.28504	0.17719
0.55	3.34193	0.44262	1.67096	0.99499	0.29521	0.18256
0.56	3.38217	0.45255	1.69109	0.99277	0.30555	0.18794
0.57	3.42252	0.46247	1.71126	0.99015	0.31606	0.19331
0.58	3.54297	0.47236	1.73149	0.98712	0.32675	0.19869
0.59	3.50357	0.48221	1.75178	0.98367	0.33762	0.20405
0.60	3.54431	0.49203	1.77215	0.97980	0.34867	0.20940
0.61	3.58522	0.50181	1.79261	0.97550	0.35991	0.21473
0.62	3.62632	0.51154	1.81316	0.97077	0.37133	0.22004
0.63	3.66764	0.52122	1.83382	0.96561	0.38294	0.22532
0.64	3.70918	0.53085	1.85459	0.96000	0.39475	0.23056
0.65	3.75098	0.54042	1.87549	0.95394	0.40676	0.23576
0.66	3.79305	0.54992	1.89653	0.94742	0.41897	0.24092
0.67	3.83543	0.55936	1.91771	0.94043	0.43140	0.24602
0.68	3.87813	0.56873	1.93906	0.93295	0.44405	0.25106
0.69	3.98119	0.57802	1.96059	0.92499	0.45693	0.25604
0.70	3.96463	0.58723	1.98231	0.91652	0.47005	0.26095
0.71	4.00848	0.59635	2.00424	0.90752	0.48342	0.26579
0.72	4.05279	0.60538	2.02639	0.98900	0.49705	0.27054
0.73	4.09758	0.61431	2.04879	0.88792	0.51097	0.27520
0.74	4.14290	0.62313	2.07145	0.87727	0.52518	0.27976
0.75	4.18879	0.63185	2.09440	0.86603	0.53719	0.28422
0.76	4.23529	0.64045	2.11765	0.85417	0.55457	0.28856
0.77	4.28247	0.64893	2.14123	0.84167	0.56981	0.29279
0.78	4.33036	0.65728	2.16518	0.82849	0.58544	0.29689
0.79	4.37905	0.66550	2.18953	0.81462	0.60151	0.30085
0.80	4.42859	0.67357	2.21430	0.80000	0.61806	0.30466
0.81	4.47908	0.68150	2.23954	0.78460	0.63514	0.30832
0.82	4.53059	0.68926	2.26529	0.76837	0.65282	0.31181
0.83	4.58323	0.69686	2.29162	0.75127	0.67116	0.31513
0.84	4.63712	0.70429	2.31856	0.73321	0.69025	0.31825
0.85	4.69239	0.71152	2.34619	0.71414	0.71022	0.32117
0.86	4.74920	0.71562	2.37460	0.64397	0.73119	0.32388
0.87	4.80773	0.72540	2.40387	0.67261	0.75333	0.33635
0.88	4.86822	0.73201	2.43111	0.64992	0.77687	0.32858